DOI: 10.18427/iri-2016-0073

Students Creating their Own Word Problem on the Basis of Given Symbolic Equation

Alenka LIPOVEC

University of Maribor, Maribor, Slovenia

alenka.lipovec@um.si

In early stage of learning, the mathematical concept is derived from the real life experience of learners. Through a long process of abstraction concept in the symbolic world is developed. We were interested in connection between prompt in form of a symbolic form of the concept and students' self-generated real situation. In other words, we analysed pupils' generalization abilities. Pupils at the age of 10 years (N = 1393) were presented a symbolic record of simple equation, where an unknown number is acting as a divisor. They were asked to produce a real life situation in the form of a word problem. Responses were categorized according to adequacy. A word problem was considered adequate, if it could be interpreted as a representation of an equation $27 : x = 9$. Only about tenth of stories were adequate. Statistically significant differences in adequacy were found regarding gender in favour of girls. Obtained stories were then analysed in terms of reality. We considered all those stories, which stem from real life situations and can actually happen in real life as realistic. The results show that more than 80 % of pupils provided realistic stories. Stories were similar to those found in mathematical textbooks. Approximately 6 % of participants provided stories with realistic context that could not be realized in meaningful situations. Approximately 10 % of participants provided word problems that included mathematical terminology (i.e. divisor, dividend, quotient...) without any realistic context. Based on these results we suggest that the mathematical lessons include several realistic mathematical stories in the phase of constructing a symbolic representation. Stories should have origin in different sources, for instance children's literature or pupils' own stories. Students could then meaningfully construct their own pictorial or symbolic representations of those situations. Additionally we believe that students should independently construct a wide range of realistic situations based on the symbolic records to strengthen the relation between real and mathematical worlds.

Introduction

Abstraction and generalization are fundamental to mathematics understanding. *Mitchelmore* (1993) proposed a model of conceptual development in mathematics consisting of two types of concepts: abstract-general concepts and abstract-apart concepts. Abstract-general concept is a product of many distinct situations and abstract-apart concept is a product of a smaller number of distinct situations.

Skemp (1976) proposed categorising relational and instrumental knowledge. This led to conceptual and procedural types of knowledge (Hiebert & LeFevre, 1986). Cognitive approaches to mathematics education equally emphasise both types of understanding/knowledge. Insight into mathematical understanding of a learner can be gained by different means, i.e. analysing learners problem solving abilities (Ellerton, 1986), learners' mathematics discourse (Sfard, 2012), learners' drawings (MacDonald, 2013), mathematical stories written by learners (Ellerton, 1986), … Mathematical stories or word problems are highly researched area (Depaepe, DeCorte & Verschaffel, 2015). We describe contextualized word problems according to Roth (1996) as "*a word problem that relate to some real-world phenomenon. A particular mathematical form (e.g., the concept of function) can model this phenomenon. When students meaningfully appropriate mathematical form by engaging with the phenomenon, the latter can be considered as a context, which elaborates the meaning of the mathematical form*" (Roth, 1996:491). The traditional way of a physical context for equations is a scale that is kept in balance.

In an older study *Lauritzen* (1992) analysed by 2nd and 6th grade students' written mathematical stories. A pedagogical experiment was performed: experimental group was set in a classroom setting rich with reading and listening of mathematical stories. At the end of that period, students were able to provide their own mathematical stories that significantly outer performed control group. Pupils stories were analysed from two perspectives: a) narration structure of a story (plot, conflict, resolving) and b) integration of mathematical concepts in stories. *Lauritzen* (1992) has argued that students write stories on the model they know.

In our research, we aimed to better understand how students translate symbolically written equation to word problems. The purpose of the study was to examine the students' knowledge in order to develop instruction that would enhance their constructions of mathematical meaning. With that in mind, we wanted to identify targets for instruction. The main goals of the study were to find out:

- whether students' generated word problems represented the required mathematical concept,
- which type of contextualization is reflected in students' generated word problems, and
- are there any gender differences according to adequacy and reality of students' generated word problems?

Methodology

Participants of our study were 1393 pupils of the 6th grade of the different elementary schools throughout Slovenia. Of these 675 were girls (48.5 %) and 718 boys (51.5 %). Average age was 11 years. The symbolic prompt was given to students in a form of symbolic equation, where an unknown number is acting as a divisor $27 : x = 9$. Participants were asked to write a real life word problem describing the equation. Equations are a part of Slovenian curriculum at the 4th and 5th grade, therefore the symbolic recording should be familiar to students. However, we have to add that mathematical education in Slovenia is predominantly procedural, i.e. students were taught how to solve the equations, less emphasis was given to the meaning of the equation. Additionally, equations with division are not as frequent as equations with addition or multiplication.

First, participants' responses were evaluated with respect to the adequacy of the word problem to represent the given equation. A description of the adequacy criteria is as follows. Word problem was considered *adequate* if the story described the equation in a mathematically meaningful way. That means main characteristic of equation should be noticeable. We were therefore looking for modelling of concepts of unknown number, equality and division. Some representations were considered *inadequate*, for instance a word problem about addition of 27 and 9 was considered inadequate. Some word problems could not be categorised according to the adequacy, therefore they were categorized as *Other* and not included in further analysis. Obtained stories were then analysed in terms of reality. Reality was considered to be *phenomenal* (a world of our actual experience), if a situation could actually happen and *semi-real* if situation could not happen. Stories outside of phenomenal (or semi real) situations were described as *non-real*. Let us illustrate that with some examples. After coding, we used a quantitative methodology and presented data using descriptive and inferential statistics.

Example 1. (Adequate) *There are 27 boys ate soccer practice. In how many groups did their trainer divided them, if there are 9 boys in each group?*

Example 2 (Inadequate) *I saw 27 boxes Samsung Galaxy Edge in a store. They sold 9 boxes. How many boxes remained at a store?*

Example 3 (Phenomenal) *Jaka played a video game. He had 3 powers. Each power killed 9 zombies. How many zombies did Jaka kill?*

Example 4 (semi-real) *Real-estate agent is selling 27 houses. How many buyers does he need if each buyer buys 9 houses?*

Example 5 (Non-real) *Number 27 is divisor and number 9 is quotient. Find the dividend.*

Results

Our first aim was to find out if students adequately represented the mathematical concept (equation). The results are given in Table 1.

Table 1. Adequate and inadequate concept regarding gender

	Adequate		Inadequate		Other		Total	
	f	f%	f	f %	f	f%	f	f %
boy	62	4.5	619	44.4	37	2.7	718	51.5
girl	97	6.9	558	40.1	20	1.0	675	48.5
Total	159	11.4	1,177	84.5	57	3.7	1,393	100.0

Table 1 shows that only a bit more than one tenth of pupils (11.4 %) wrote a mathematical word story that could be interpreted as contextualization of equation $27 : x = 9$. Modelling with scale was not noticed at all. A vast majority of pupils (84.5 %) wrote stories where other mathematical operations (like multiplication or subtraction) were prevalent. Only those participants for whom adequacy could be determined (N=1336) were confronted regarding gender differences. Yates (continuity) correction yield to χ^2=9.827, P=0.002. Statistically significant differences were shown in favour of girls regarding adequacy of concept included in mathematical story.

In next step, we analysed stories regarding reality. The results of reality regarding gender are given in Table 2.

Table 2. Phenomenal, semi-real and non-real stories regarding gender

	phenomenal		Semi real		Non-real		Total	
	f	f%	f	f %	f	f%	f	f %
boy	557	41.7	47	3.5	77	5.8	681	51.0
girl	573	42.9	34	2.5	48	3.6	655	49.0
Total	1130	84.6	81	6.0	125	9.4	1,336	100.0

Phenomenal stories prevailed in more than 80 %. We have to add that those stories could represent other concepts besides equation. Again girls were statistically significant better in providing "real life" (phenomenal) stories (χ^2=8.538, P=0.014)

Discussion

The research shows that difficulties with division (Huber, Fischer, Moeller & Numerk, 2013) and algebraic notions like unknowns in equations (Susac, Bubic, Vrbanc & Planinic, 2014) can be persistent even at secondary school, even if the task is purely procedural. According to our results, difficulties could be observed also on the conceptual type of knowledge. Even though more than 80 % of students provided a real life word problem, only a tenth of those problems accurately described the

given equation. Students generated word problems in our data were very similar to those in Slovenian textbooks. Our results confirm *Mitchelmore* (1993) suggestion that most mathematics teaching tends to produce abstract-apart concepts.

Our results also resemble to the results of study conducted by *Winograd* (1991). He argues that students' generated word problems begin with a description of the general theme of the problem, followed by concretization of the problem and integration of the data that is important to solve the problem. Almost all word problems from our data set followed that pattern.

Gender differences favouring girls regularly occurred in our results. Slovenian international comparisons on mathematical TIMSS (IEA, 2011) and PISA (OECD, 2012) do not show gender differences at mathematical achievements; however, literacy related study PIRLS (IEA, 2011) show slight preference for girls, what is also visible in our results.

Conclusions

Possible form of representing mathematical concept is also a real life situation or contextualized word problems. Word problems give mathematics, that they model, a meaning. They connect mathematics with pupils' experiences and allow students' to relate context of a mathematical word problem to the real world phenomena.

Often times, students are asked to solve only problems they see in their textbook or that have been written by their teacher. On the other hand, students' self-generated stories reflect personal experience that students bring from their own lives what make problems more meaningful to students.

The results of reported study show that Slovenian 6th grade students do not perform well on writing word problems when prompt is a symbolic equation with division. We propose including similar assignments in early mathematical learning when concepts are easier to contextualize and inclusion of children's literature in mathematical lessons.

References

Depaepe, F., DeCorte, E., & Verschaffel, L. (2015). Students' non realistic mathematical modelling as a drawback of teachers' beliefs about and approaches to word problems. In B. Pepin, & B. Roesken-Winter (Eds.), *From beliefs to dynamic affect systems in mathematics education: Exploring a mosaic of relationships and interactions* (pp. 137-156). Heidelberg; New York; Dordrecht; London: Springer.

Ellerton, N. F. (1986). Children's made-up mathematics problems — a new perspective on talented mathematicians. *Educational Studies in Mathematics,* 17 (3), 261-271.

Hiebert, J., & LeFevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge:The case of mathematics* (pp. 1-27). Hillsdale: Lawrence Erlbaum Associates.

Huber, S., Fischer, U., Moeller, K., & Numerk, H. C. (2013). In the interrelation of multiplication and division in secondary school children. *Frontiers in Psychology*, 4, 1-10.

IEA. (2011). *TIMSS & PIRLS 2011*. Retrieved from http://timssandpirls.bc.edu/timss2011/international-database.html [04.05.2016].

Lauritzen, C. (1992). When children write math stories. In *West Regional Conference of the International Reading Association* (pp. 1-11). Portland. Retrieved from http://files.eric.ed.gov/fulltext/ED396304.pdf [04.05.2016].

MacDonald, A. (2013). Using children's representations to investigate meaning-making in mathematics. *Australasian Journal of Early Childhood*, 38 (2), 65-73.

Mitchelmore, M. (1993). Abstraction, generalization and conceptual change in mathematics. *Hiroshima Journal of Mathematics Education,* (2), 45-57.

OECD. (2012). *Programme for international student assessment*. Retrieved 4. 5 2016 from http://www.oecd.org/pisa/ [04.05.2016].

Roth, W. M. (1996). Where is the context in cotextual word problems? Mathematical practicec and products in grade 8 students' answers to story problem. *Cognition and Instruction,* 14 (4), 487-527.

Sfard, A. (2012). Research problems in mathematics education revisited: New clothes – and no emperor. *For the learning of mathematics,* 32 (2), 3-5.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.

Susac, A., Bubic, A., Vrbanc, A., & Planinic, M. (2014). Development of abstract mathematical reasoning: The case of algebra. *Frontiers in Human Neuroscience*, 1-10.

Winograd, K. (1991). Writing, solving and sharing original math story problems: Case study of fifth grade childrens' cognitive behavior. In *Annual Meeting of the American Educational Research Asociation* (pp. 1-45). Chichago.